Equilibration of symmetric instability and inertial oscillations at an idealised submesoscale front

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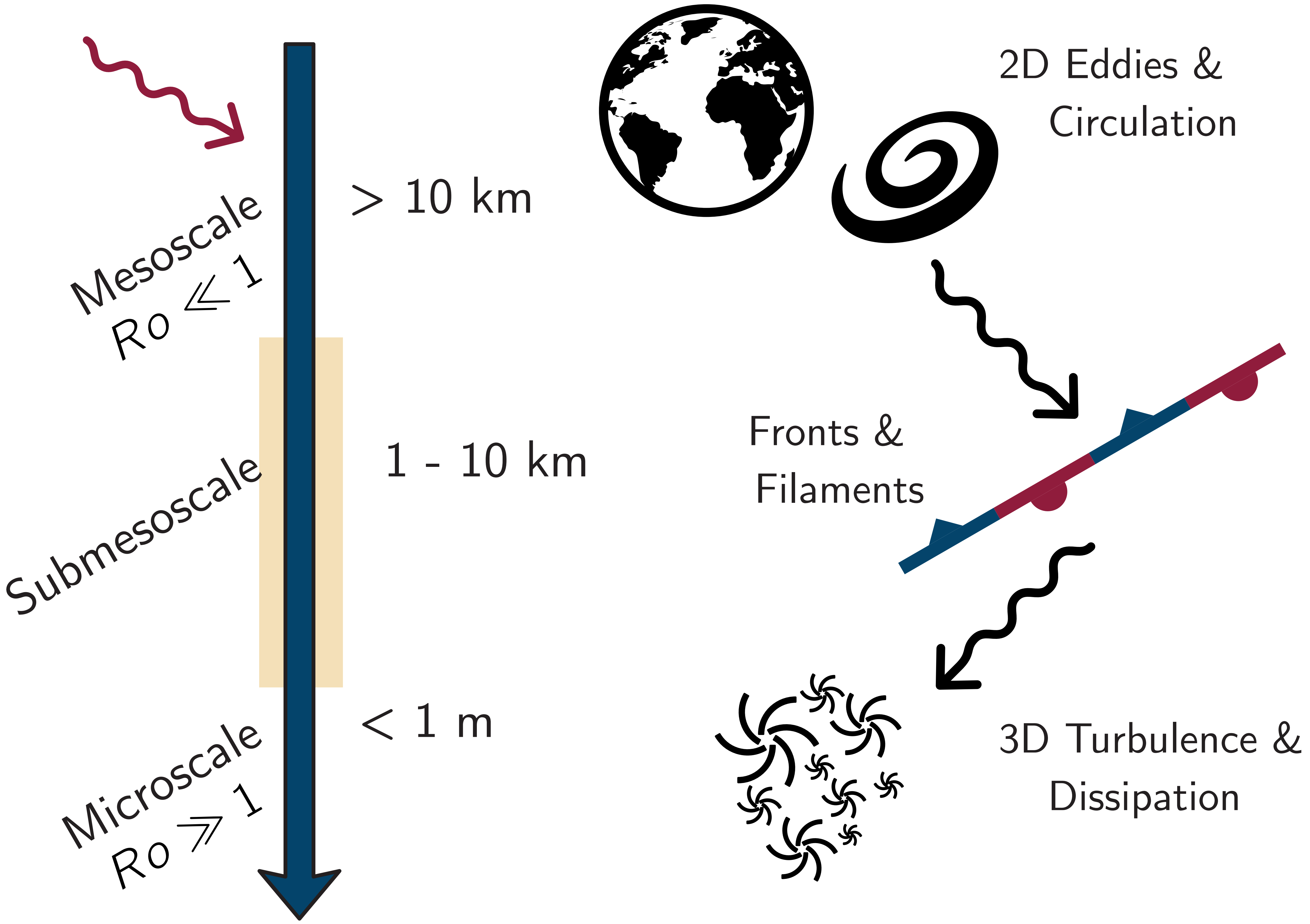
Motivation: Submesoscale Frontal Regions



Front: A region with a large horizontal buoyancy gradient, $M^2 \equiv \partial_x \bar{b}$, typically in near-geostrophic balance.

- Sloping isopycnals and outcropping in fronts encourage exchange with the ocean interior.

Fronts: The Gulf Stream

$M^2 \equiv \partial_x \bar{b}$

f: Coriolis Parameter

$M^2/f^2 \approx 40$

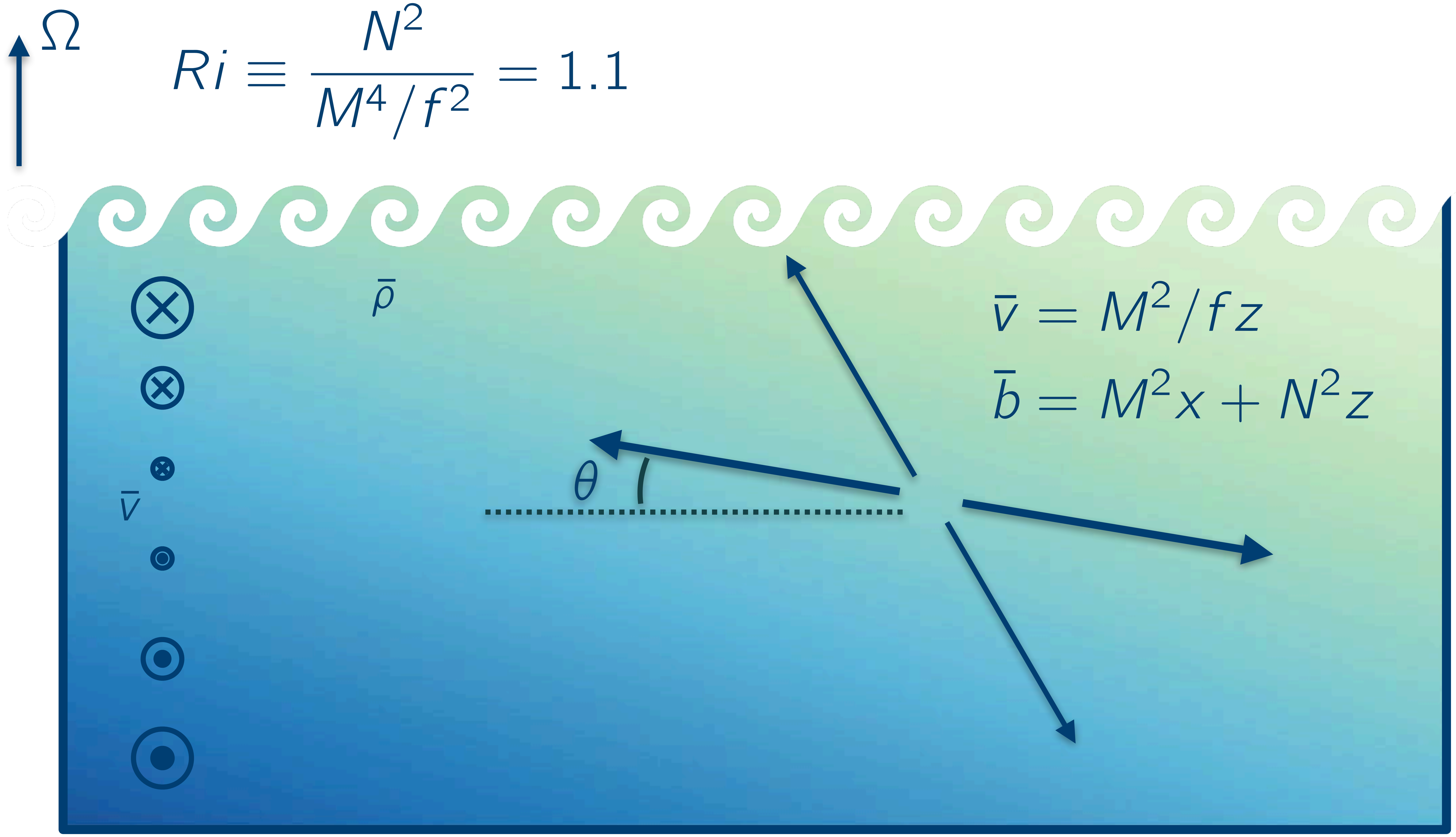
NASA/USGS

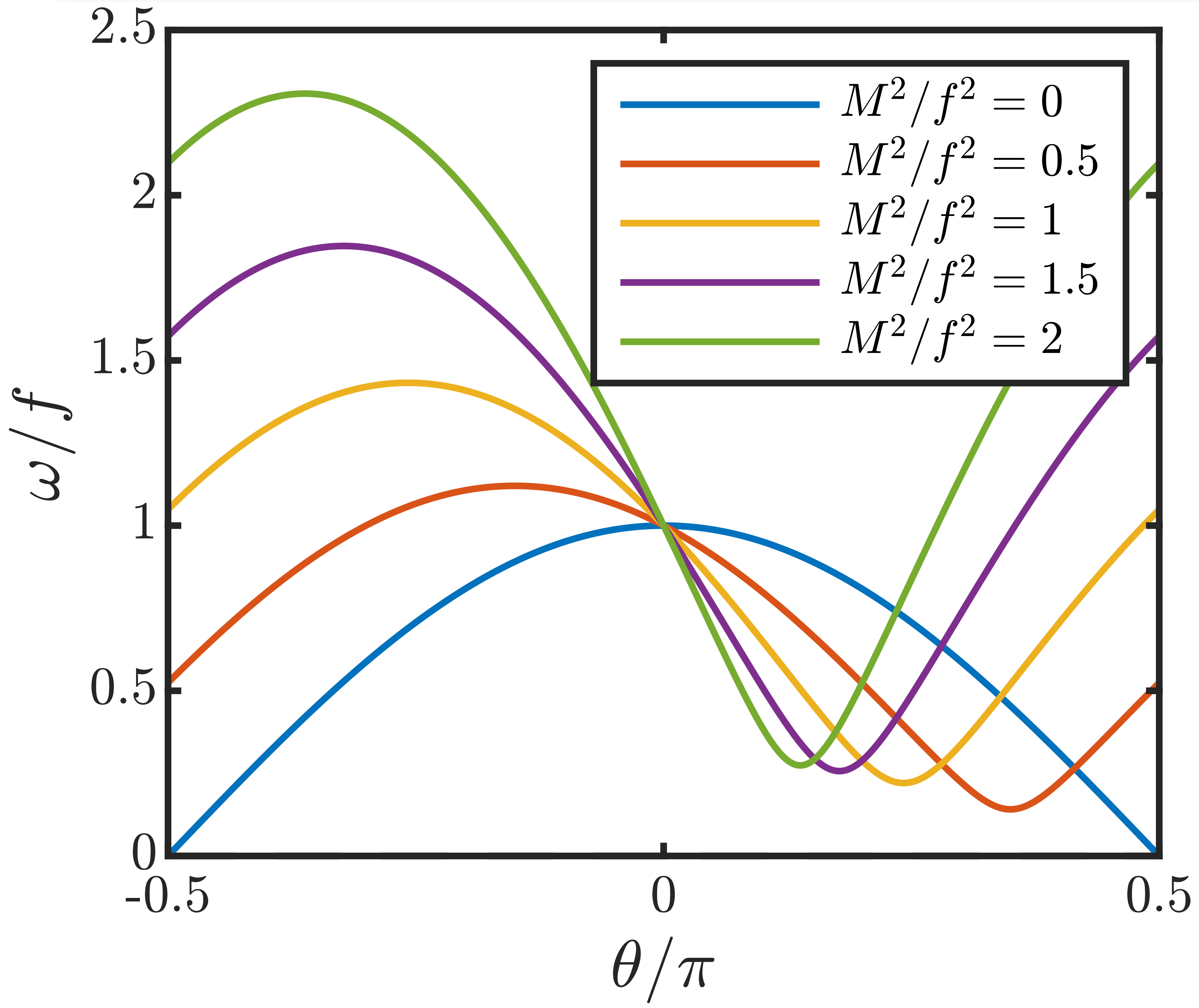
Fronts: River Outflow into the Gulf of Mexico

Inertia-Gravity Waves

$$Ri \equiv \frac{N^2}{M^4/f^2} = 1.1$$

$$\omega^2 = f^2 \frac{k_z^2}{|\mathbf{k}|^2} - 2M^2 \frac{k_x k_z}{|\mathbf{k}|^2} + N^2 \frac{k_x^2}{|\mathbf{k}|^2}$$

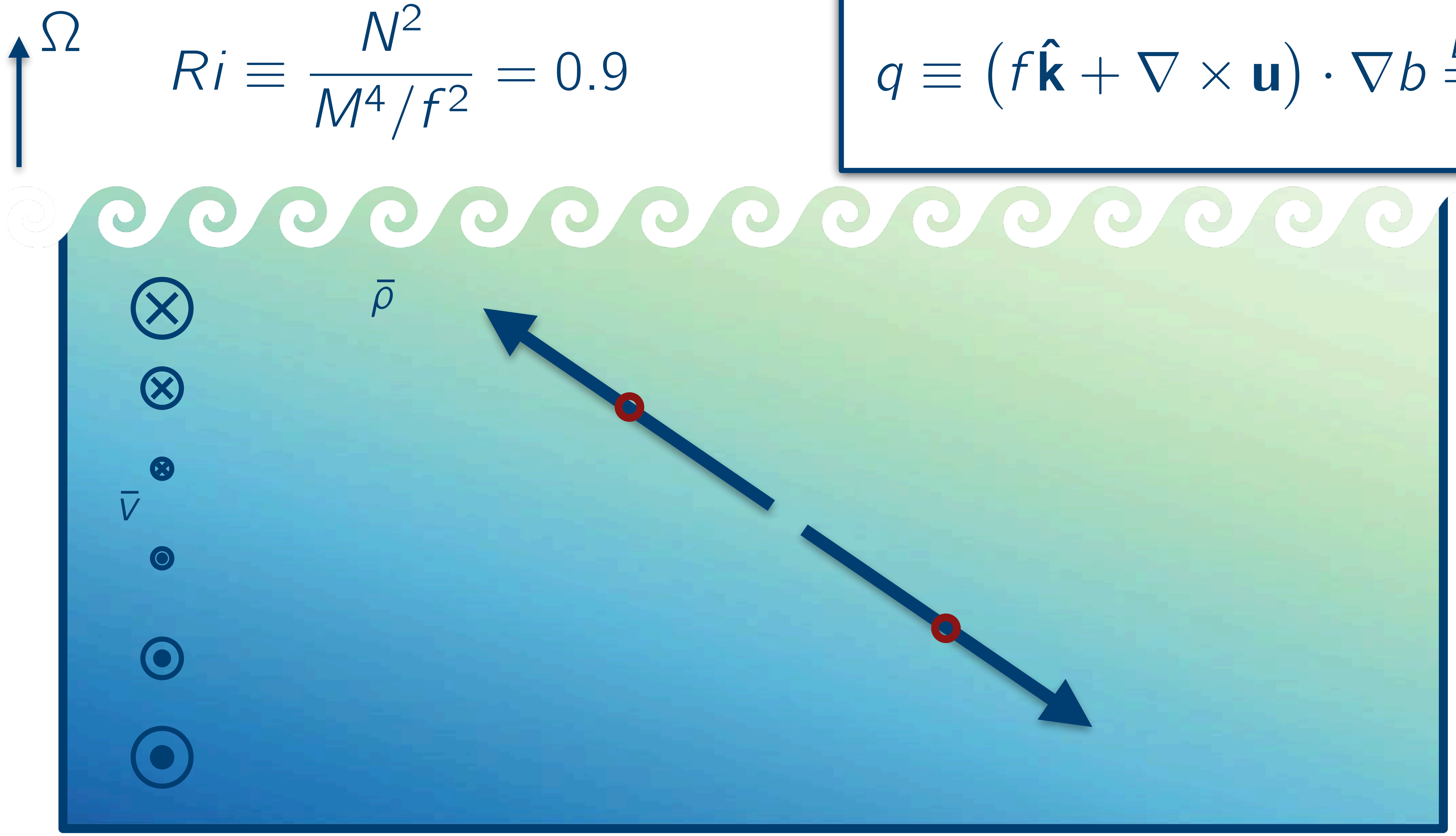


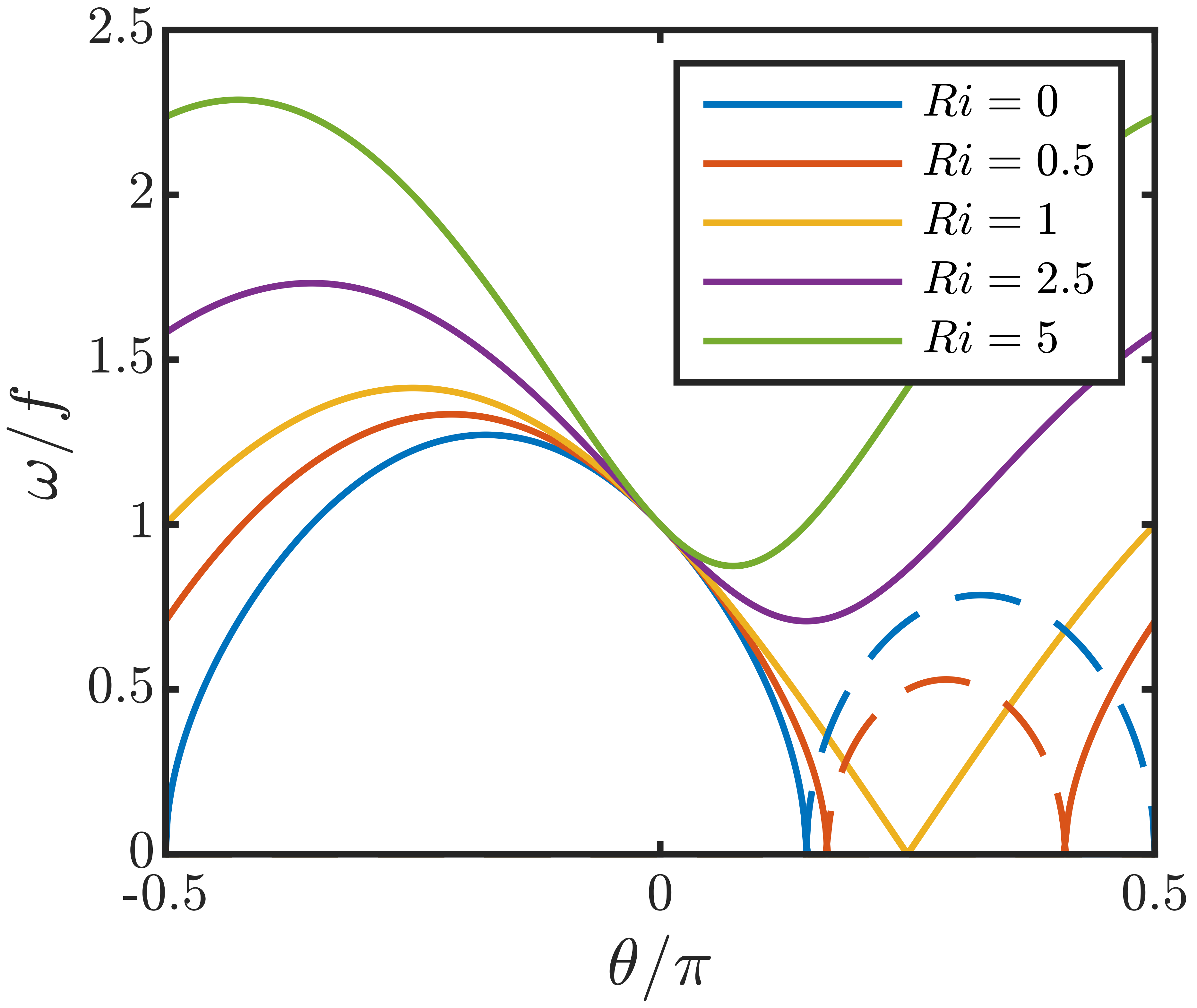


Inertia-Gravity Waves → Symmetric Instability (SI)

$$Ri \equiv \frac{N^2}{M^4/f^2} = 0.9$$

$$q \equiv \left(f\hat{\mathbf{k}} + \nabla \times \mathbf{u}\right) \cdot \nabla b \overset{B}{=} f N^2 \left(1 - \frac{1}{Ri}\right) < 0$$

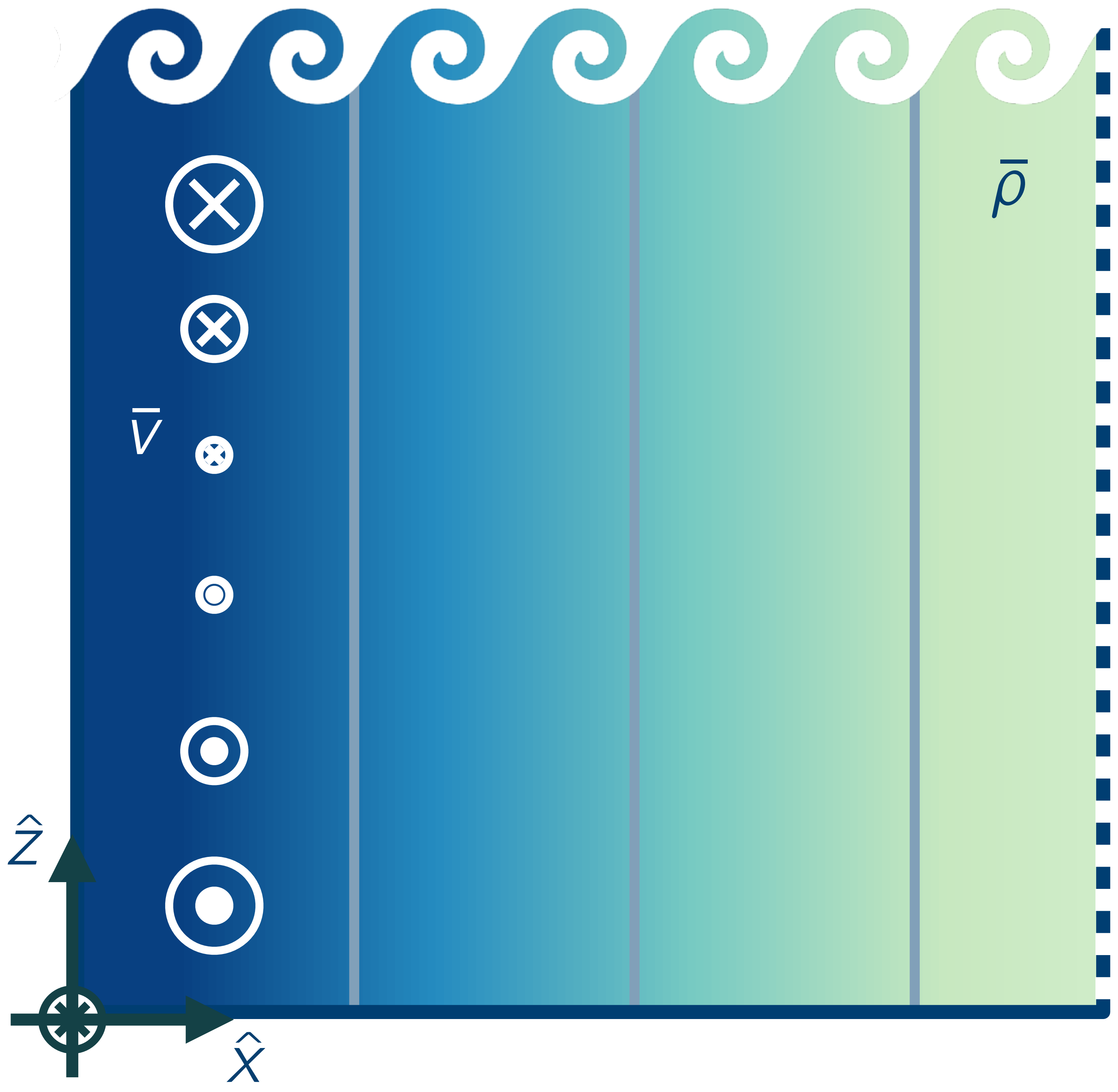




Equilibration of an SI-Unstable Front

$$\bar{v} = M^2/fz$$
$$\bar{b} = M^2x + N^2z$$

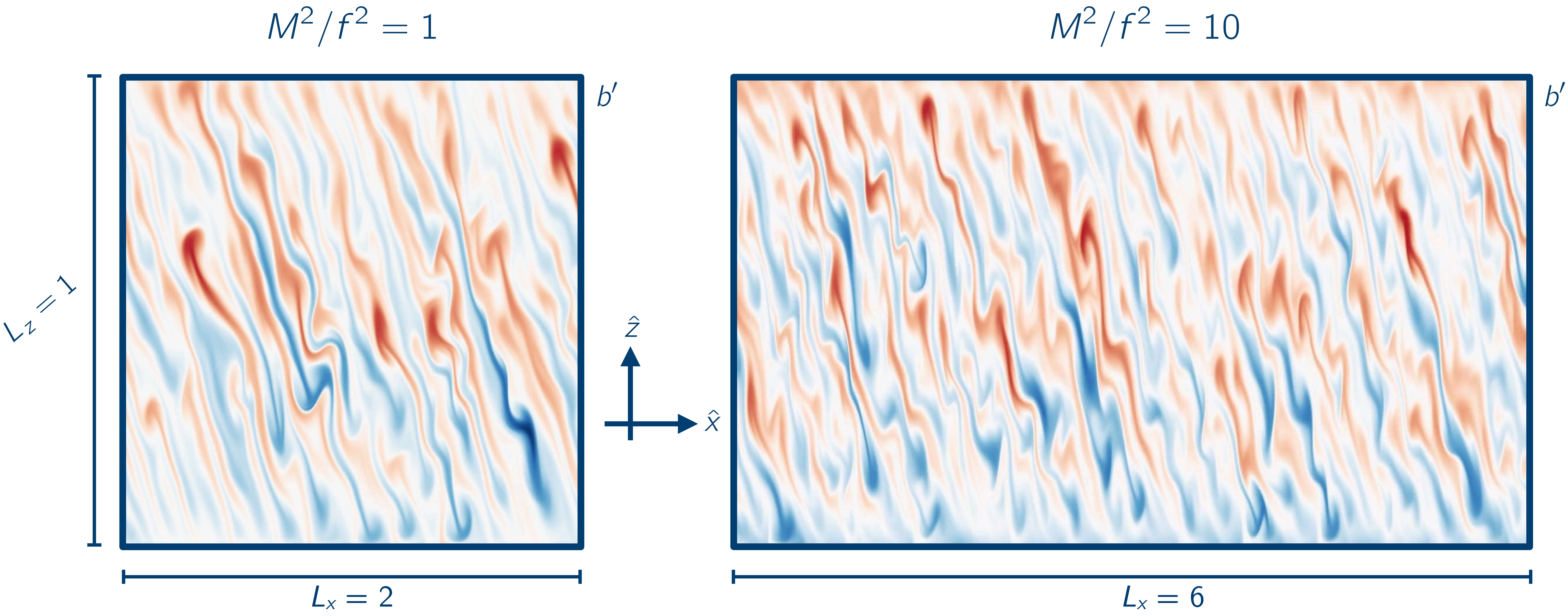
- 2D pseudo-spectral / finite difference simulations
  - Horizontal periodicity in perturbations from $\bar{b}$
  - Initial condition in Thermal Wind Balance
- Focus on $Ri = 0$ (i.e. "vertical" front) and $Re = 10^5$
- SI can mix down the Thermal Wind Shear



Primary Aim: Examine the dependence of the equilibration of an SI-unstable front on front strength, M^2/f^2.

Eady (1949)

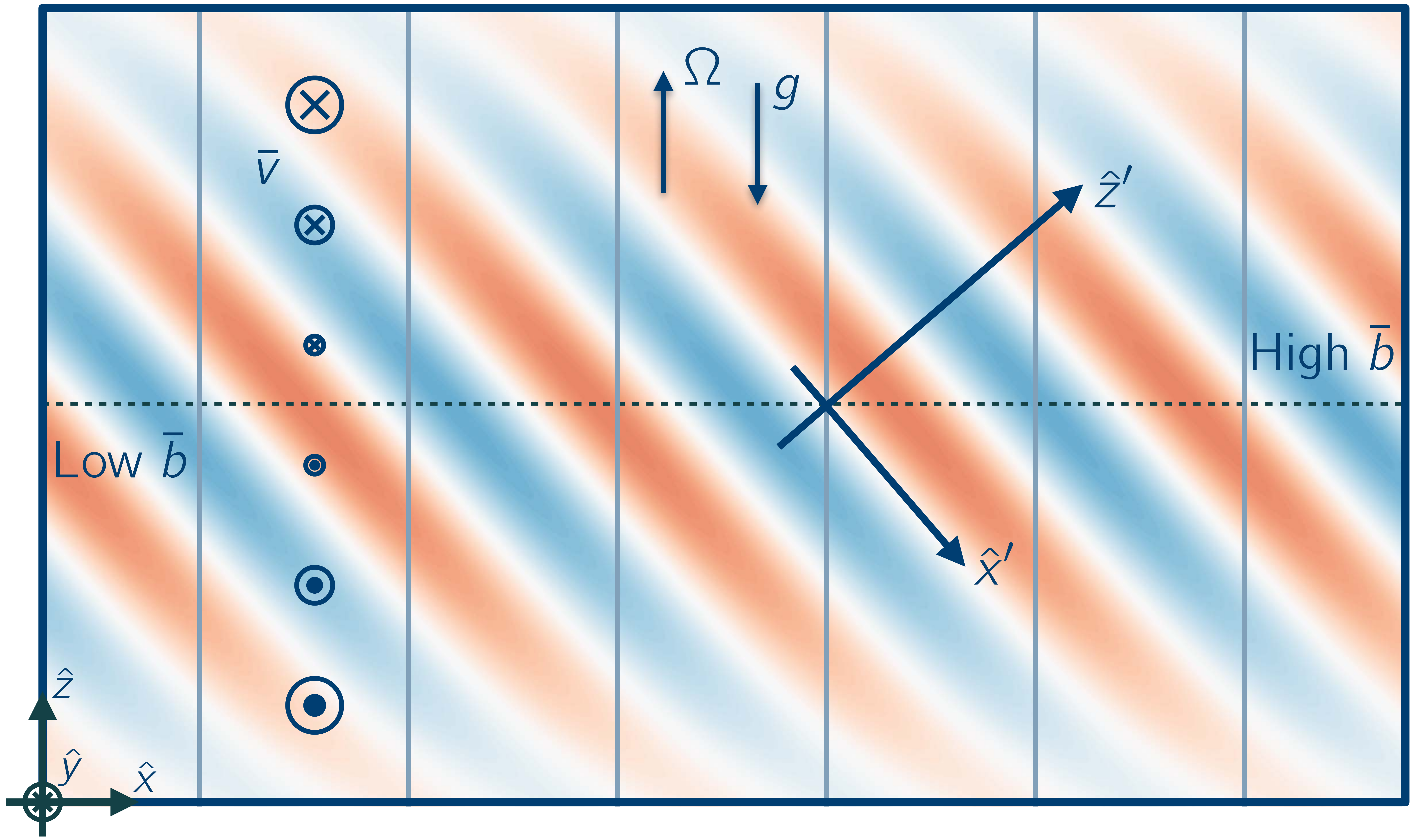
Equilibration of Frontal Regions



Secondary Linear Stability: Kelvin-Helmholtz

- 1D pseudo-spectral solver in rotated coordinates
- Criticality condition:

$$\sigma_{\mathrm{KH}} = \sigma_{\mathrm{SI}} \text{ at } \tau_c$$

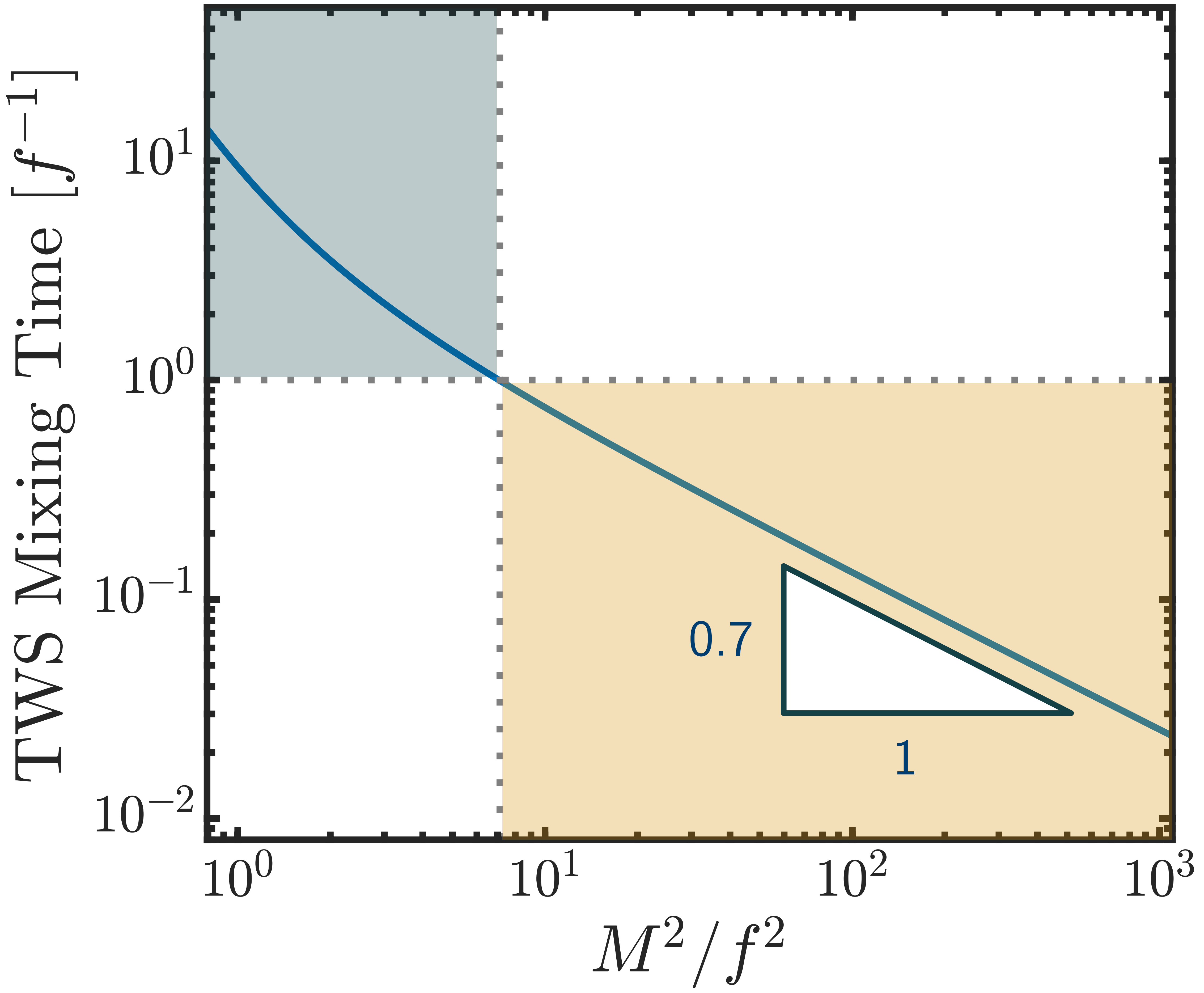


Cumulative Transport by SI

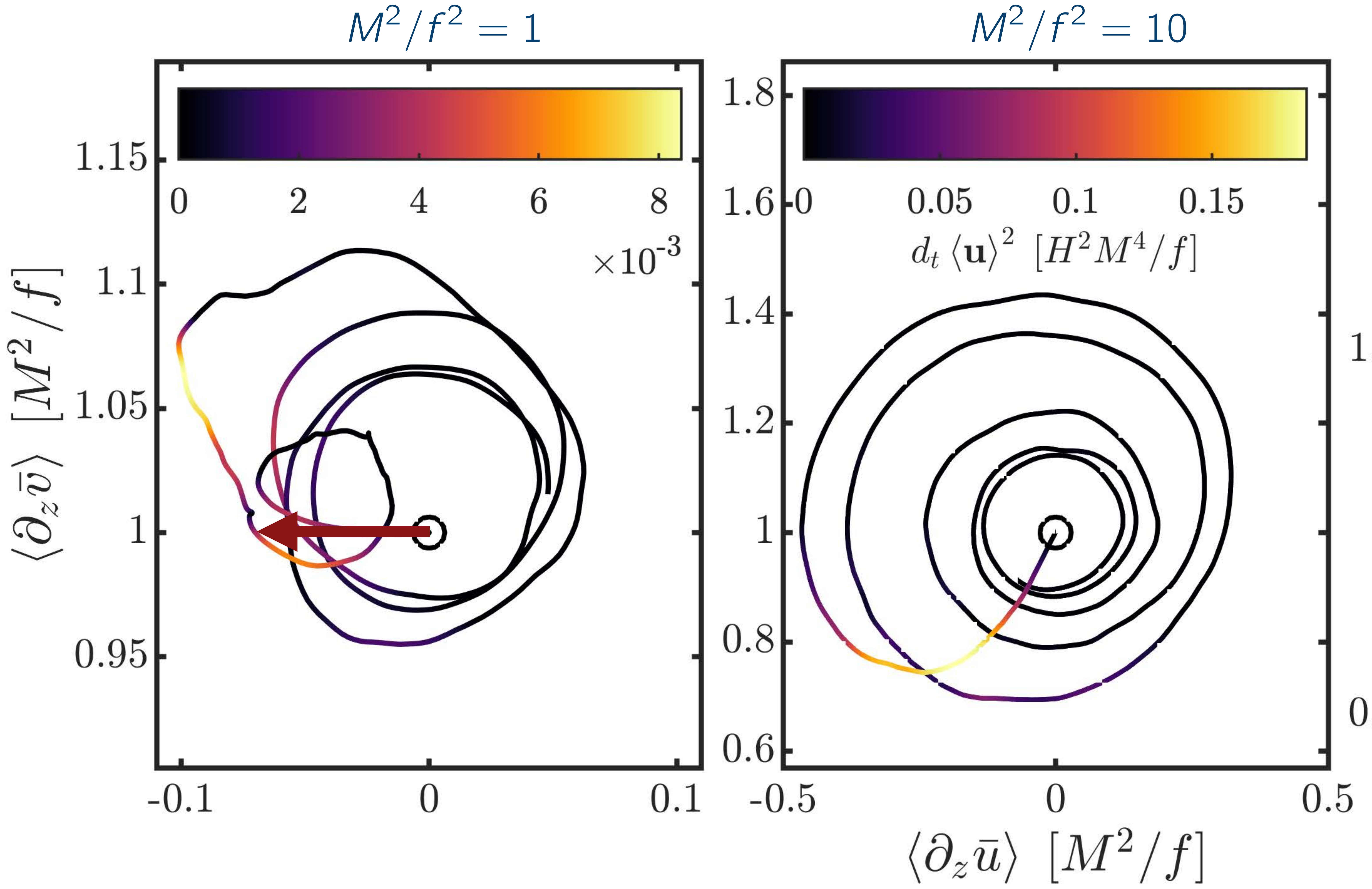
$$\frac{\partial}{\partial t}\left(\frac{\partial \bar{v}}{\partial z}\right) = \frac{\partial^2 \overline{v'w'}}{\partial z^2} - f\frac{\partial \bar{u}}{\partial z}$$

- Cumulative contribution of SI modes to the transport, through τ_c
- Difference in adjustment behaviour depending on the Shear Forcing vs. Inherent Time-scale

Shear Forcing: $\frac{M^2/f}{\partial_z^2 \overline{v'w'}}$ vs. Inherent: f^{-1}



Frontal Response: Slumping vs Falling



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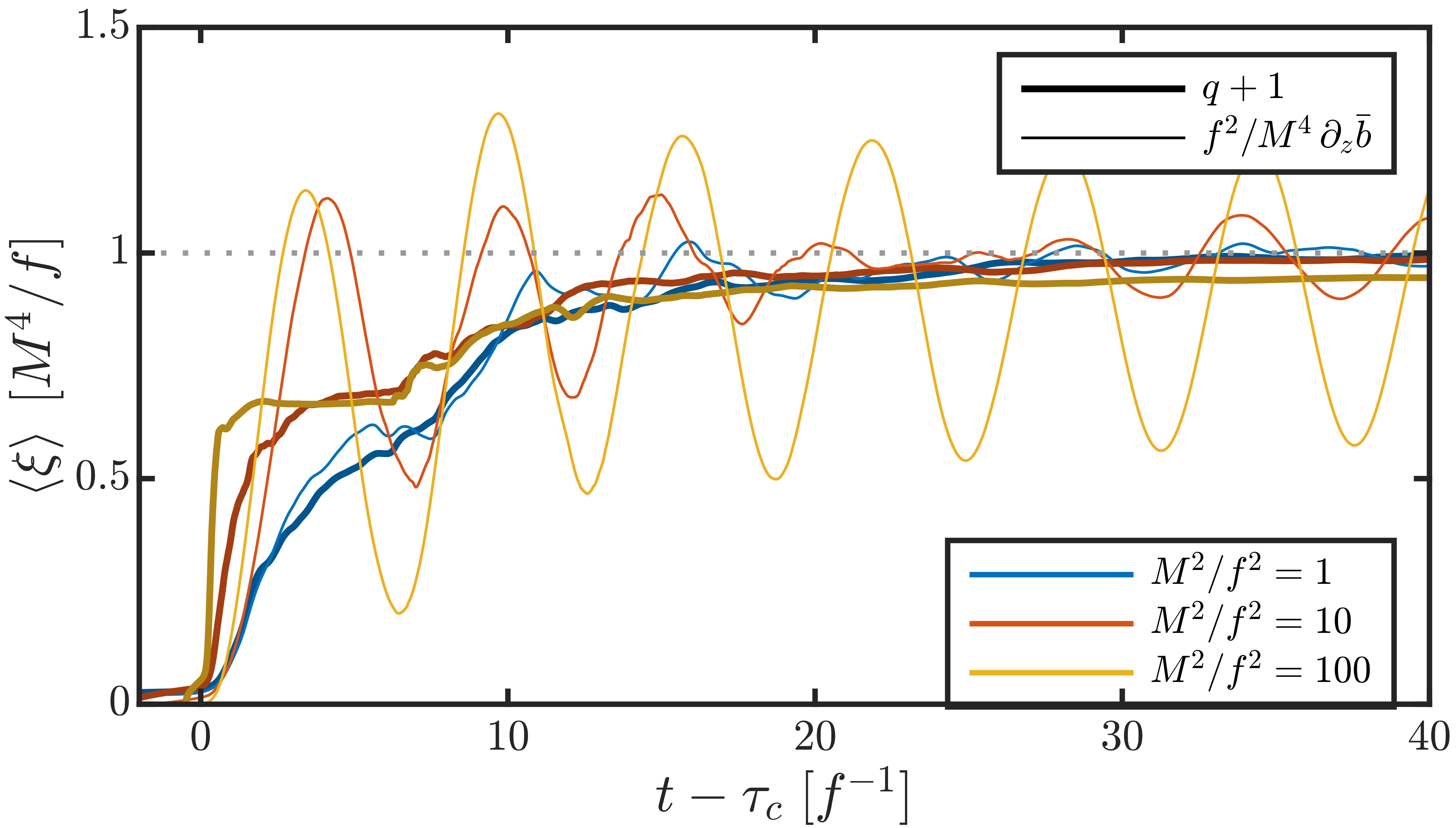
Damping to Equilibrium

- PV fluxes stabilise the front
  ∴ Good metric for equilibration
- SI-turbulence ⇄ PV Fluxes
  seems to be self-regulatory
- Constant damping time-scale,
  $\tau_q f \approx 5.5$

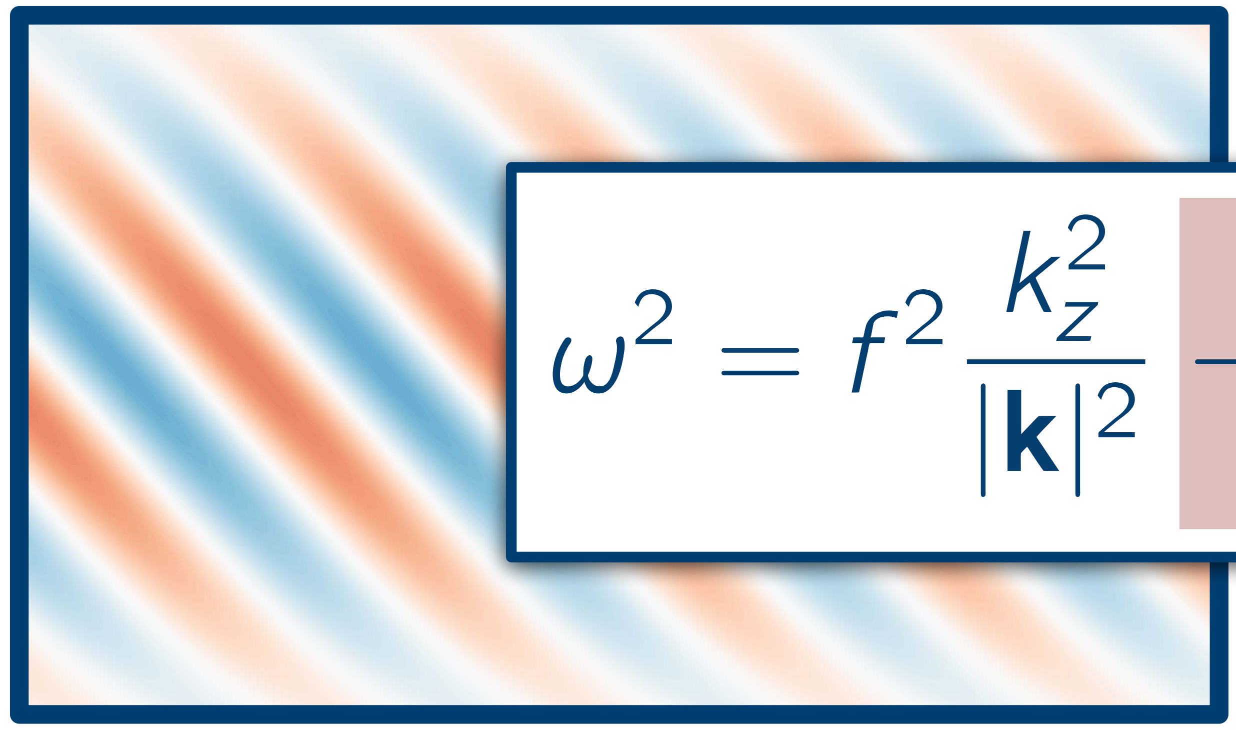


Conclusions

- SI can transport significant geostrophic momentum prior to transition.
- The adjustment response and equilibration of the front depends strongly on the front strength, M^2/f^2.
  - **Weak fronts** slowly slump into equilibrium
  - **Strong fronts** suddenly lose balance and undergo large oscillations

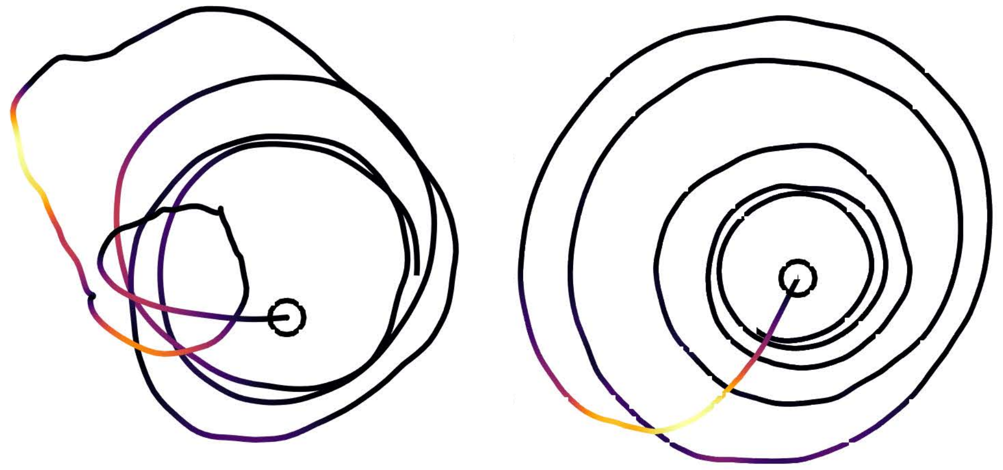
Symmetric Instability

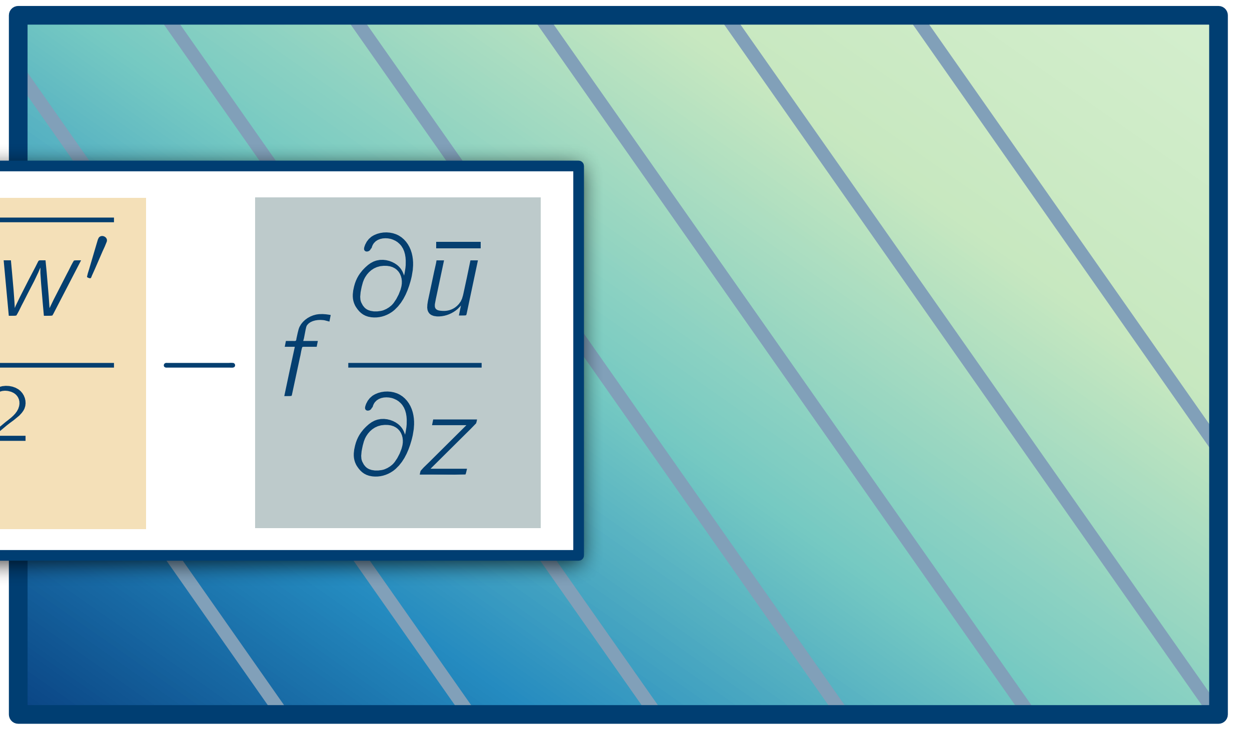
$q < 0$



$$\omega^2 = f^2 \frac{k_z^2}{|\mathbf{k}|^2} - 2M^2 \frac{k_x k_z}{|\mathbf{k}|^2} + N^2 \frac{k_x^2}{|\mathbf{k}|^2}$$

Geostrophic Adjustment

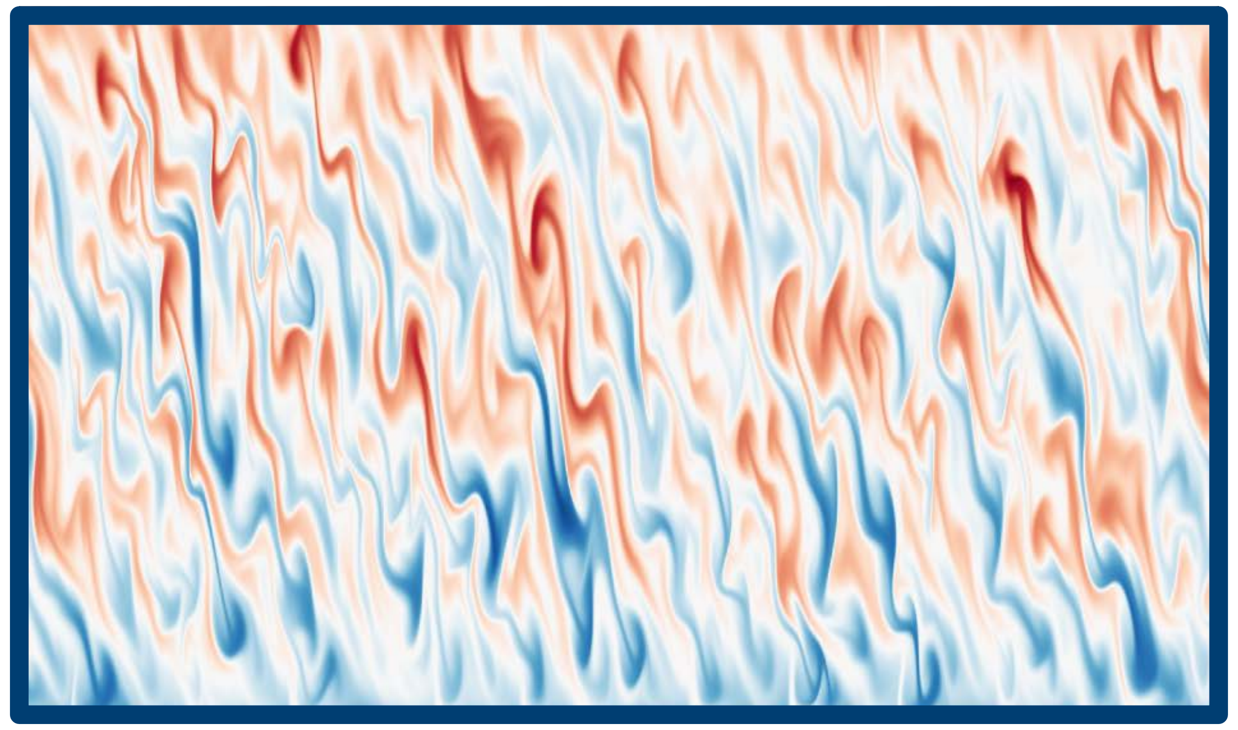


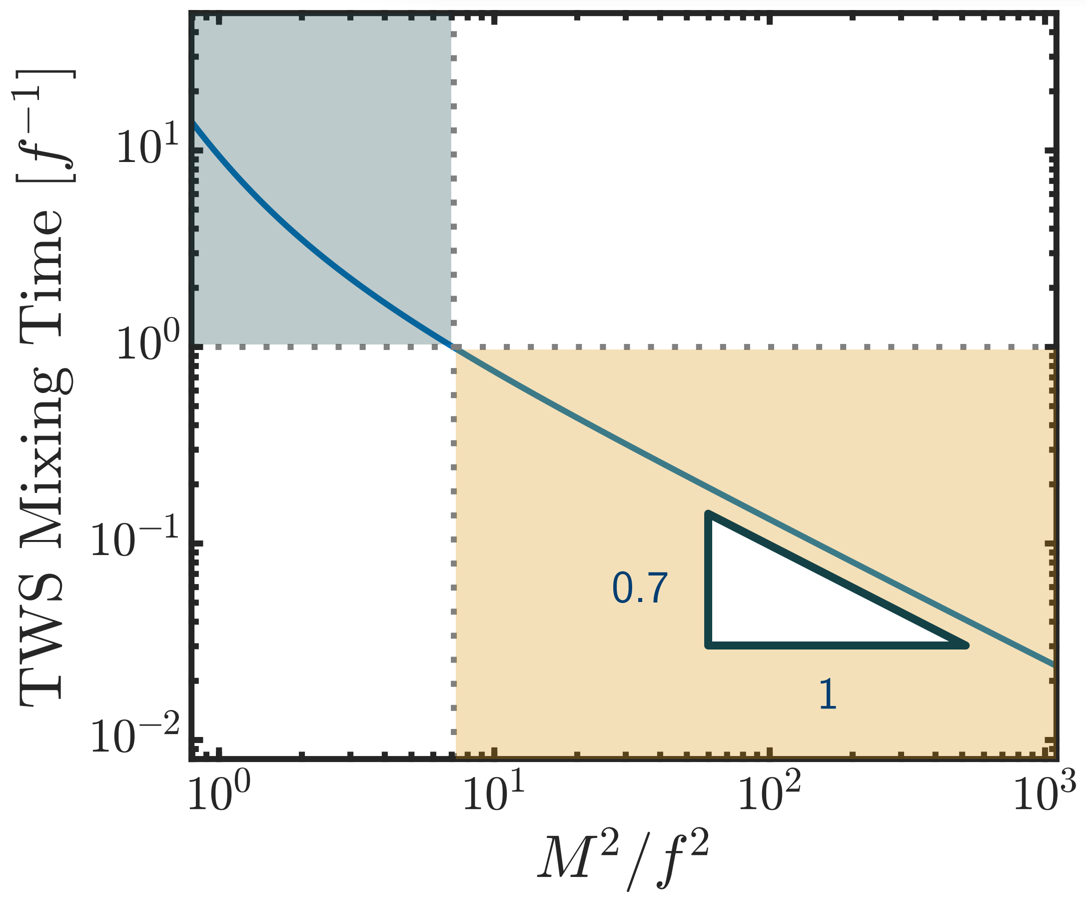


$$\frac{\partial}{\partial t}\left(\frac{\partial \bar{v}}{\partial z}\right) = \frac{\partial^2 \overline{v'w'}}{\partial z^2} - f\frac{\partial \bar{u}}{\partial z}$$

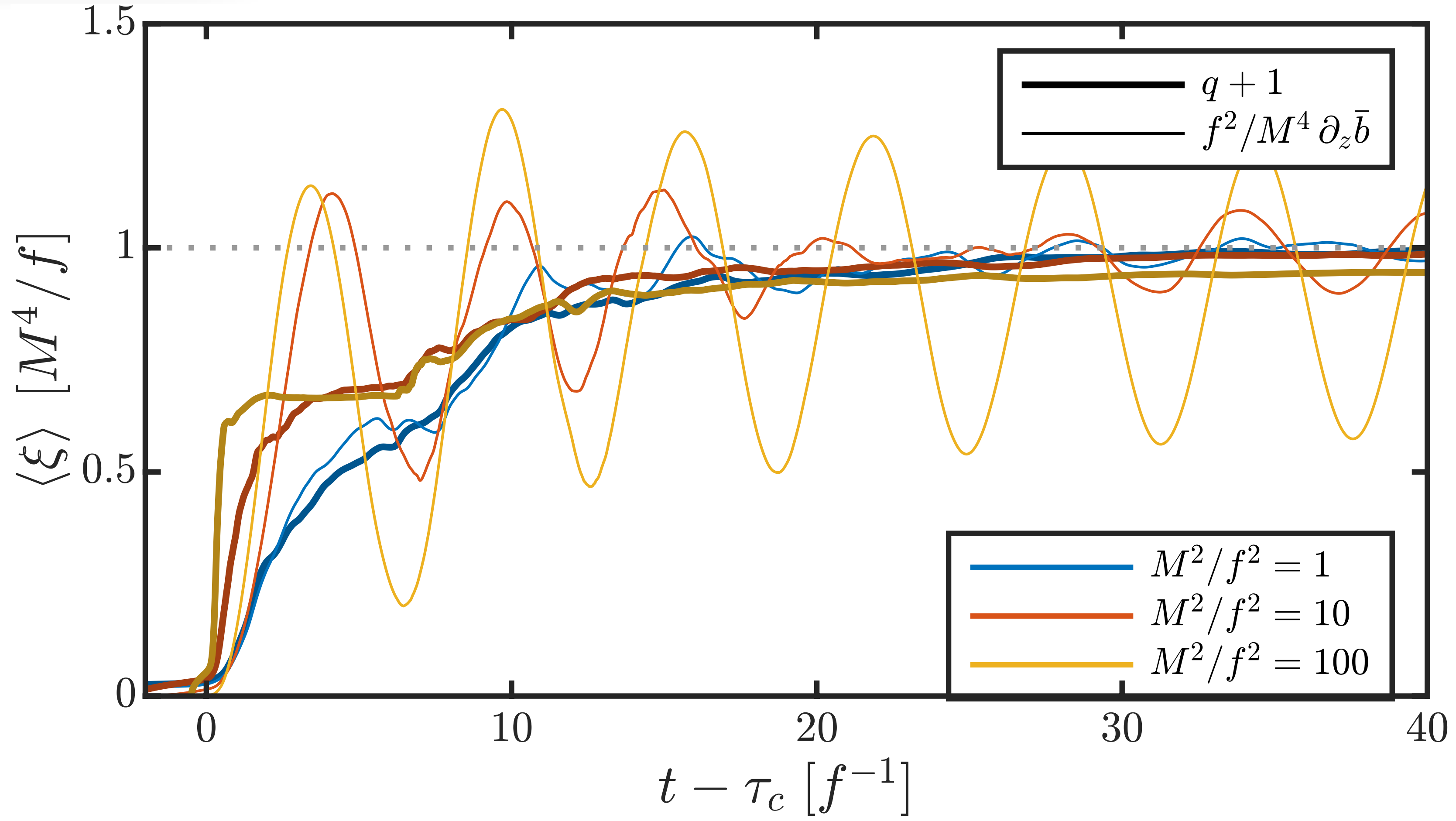
Questions?

Secondary Instability





Equilibration



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