Transport and evolution of symmetric instability in an unstratified layer

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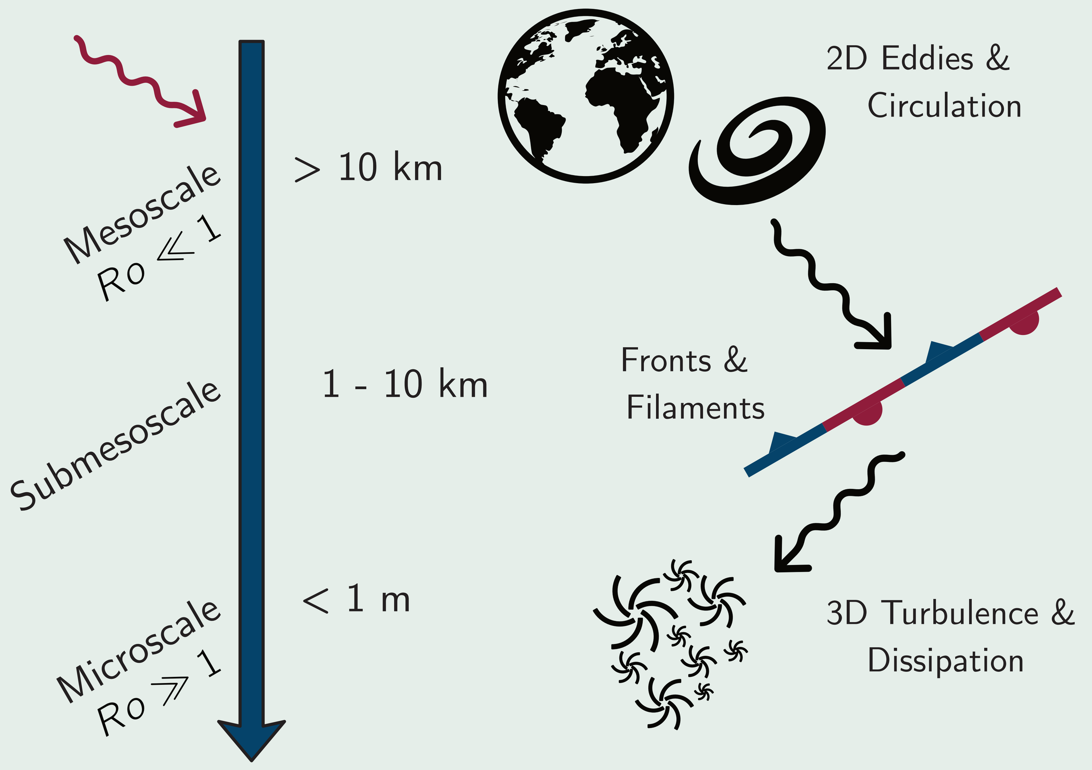
1 Motivation

What sorts of processes occur in submesoscale frontal regions, often on the sub-gridscale of GCMs?

How is energy at mesoscales mediated by submesoscale dynamics to continue the turbulent energy cascade?



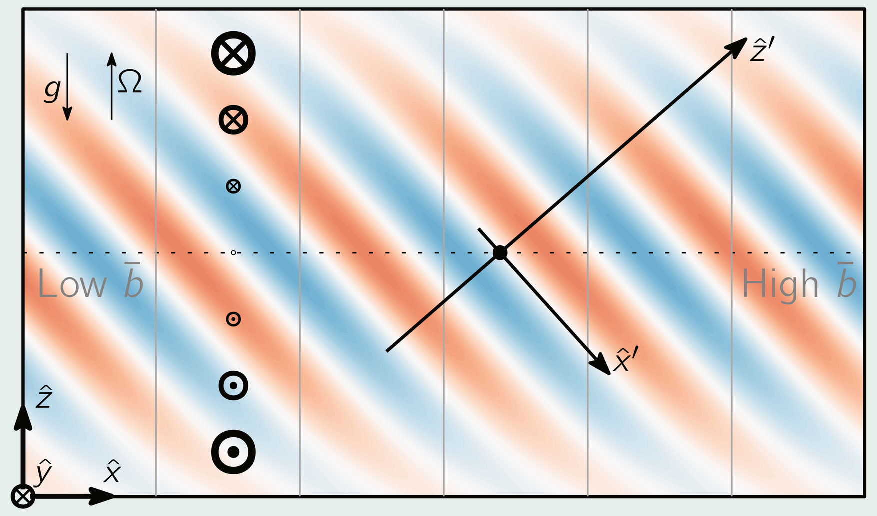
- Much of the energy in the ocean exists in mesoscale structures and is constrained by rotation & stratification to be nearly horizontal (2D).
- Instabilities active in submesoscale frontal regions:
  - **Inertial Instability:** When $\omega_z < -f$
  - **Kelvin-Helmholtz (KH) Instability**: Possible if $Ri < 1/4$
  - **Baroclinic Instability:** When $Ri > 0$
    ↳ Mode structure is *along* the front
  - **Symmetric Instability (SI):** When $qf < 0$
    ↳ Mode structure is *across* the front
    - Dominant in the fronts considered, with $Ro \gtrsim 1$
- Vertical fronts can be generated in the wake of mixing events (storms) or consistently in shallow coastal regions with a freshwater source.

2 Linear Symmetric Instability

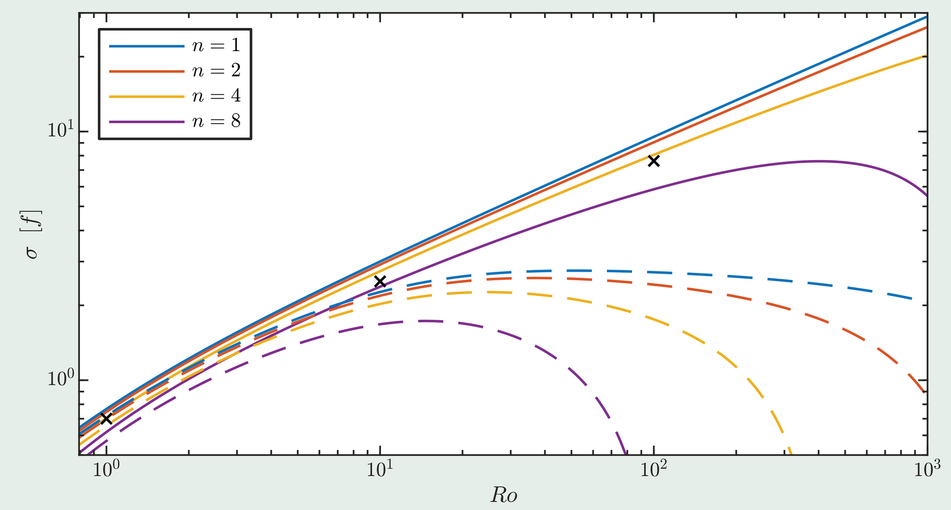
- Symmetric Instability is a form of stratified inertial instability which can occur when the Ertel potential vorticity, q, is of the opposite sign to the Coriolis parameter (i.e. $qf < 0$).
- Relevant parameters:
  - Rossby Number: $Ro \equiv M^2/f^2$
    i.e. the strength of lateral stratification
  - Reynolds Number: $Re \equiv \dfrac{H^2M^2}{f\nu}$
  - Richardson Number: $Ri \equiv N^2f^2/M^4 \to 0$
- Eady Problem Basic State (in Thermal Wind Balance):

$$\nabla\bar{b} = M^2\,\hat{\mathbf{x}} \qquad \nabla\bar{v} = M^2/f\,\hat{\mathbf{z}}$$

2 Symmetric Instability (cont.)



- Semi-analytic solution for viscous, vertically-bounded, & unstratified fronts without the hydrostatic approximation:

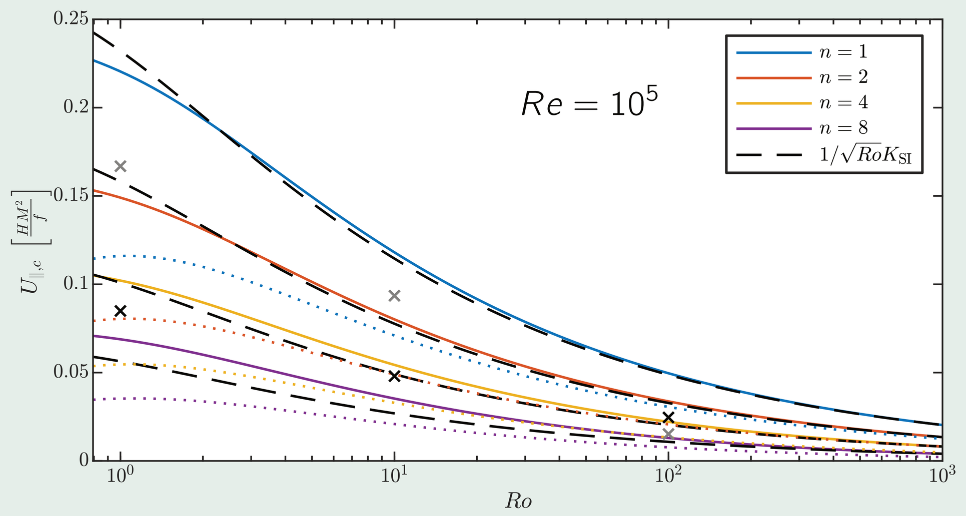


- Non-hydrostatic effects regularise the singularity known from literature in the vertical limit, i.e. as $Ri \to 0$.

3 Secondary Shear Instability

When does SI break down at finite amplitude?

- 1D linear Kelvin-Helmholtz stability problem of a sinusoidal mode is superposed on the rotated Eady basic state to find the critical time, τ_c when SI & KH growth rates are equal.

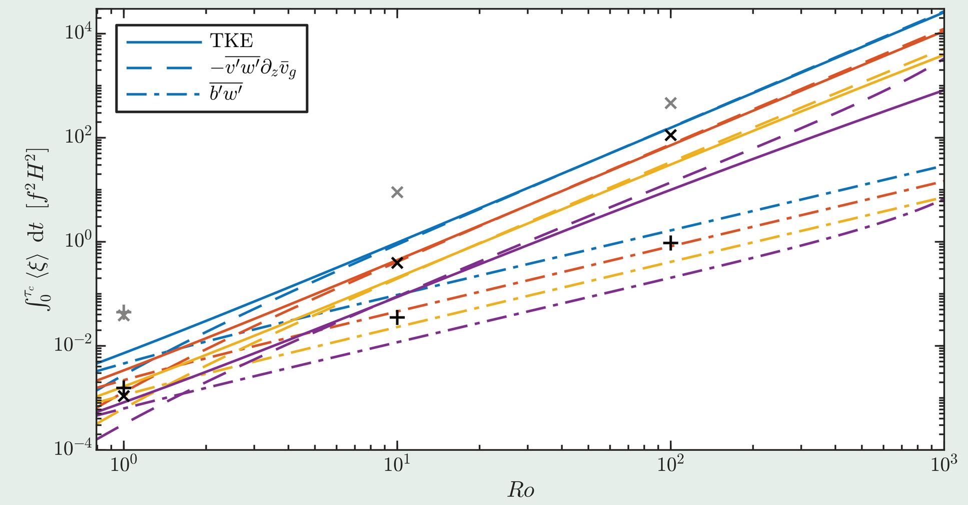


- Basic Kelvin-Helmholtz theory suggests a critical velocity amplitude at $t = \tau_c$ of $U_{0,c} \propto \sqrt{Ro}/K_{SI}$.
- $Ro_{KH} \propto \sqrt{Ro}$ and so for $Ro \sim 1$, non-traditional KH effects (rotation & lateral stratification) cannot be ignored.

4 Linear Mode Transport

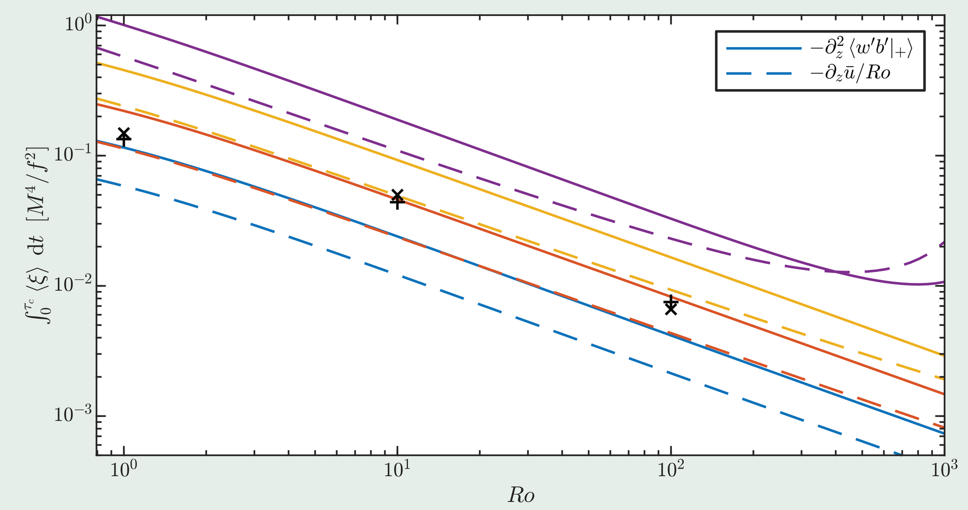
What effect does the growing Symmetric Instability have on the density and momentum structure of the front?

- At finite amplitude, the SI modes are able to generate non-negligible transport of buoyancy and geostrophic momentum, as shown below by time-integrating through τ_c.

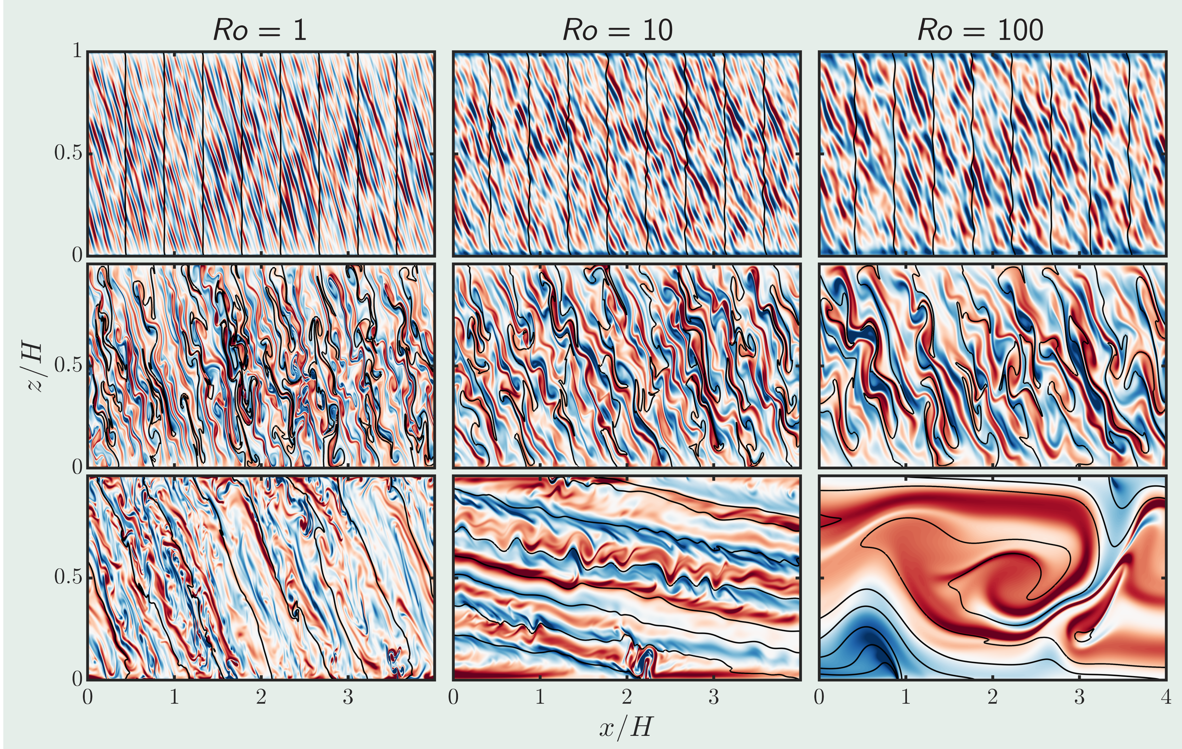


- Linear modes are able to rearrange the mean stratification,

$$\partial_t N^2 = -Ro^{-1}\partial_z\bar{u} - \partial_z^2\overline{w'b'}$$



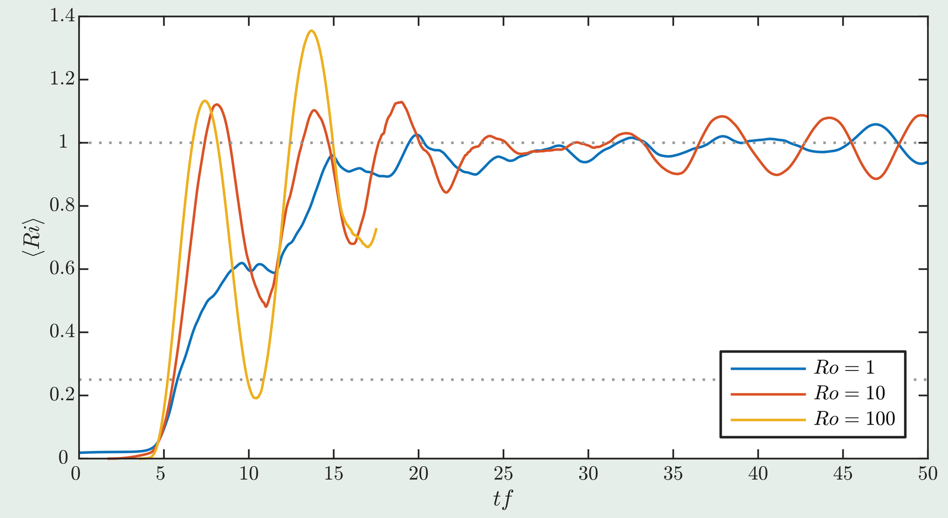
- As Ro increases, the restratification as measured by $Ri(\tau_c)$ (above) decreases, and is small compared to the value for a balanced SI-neutral state (i.e. $Ri = 1$).
  ↳ Indicates the relative importance of Symmetric Instability, versus Kelvin-Helmholtz & turbulence, to the equilibration.
  ↳ Agrees with the Taylor & Ferrari (2009) results showing KH instability & turbulence as critical to equilibration.
- Nonlinear simulation showing the linear & adjustment phases:



5 Adjustment & Equilibration

What influence do the primary and secondary instabilities have on the nonlinear evolution and equilibration?

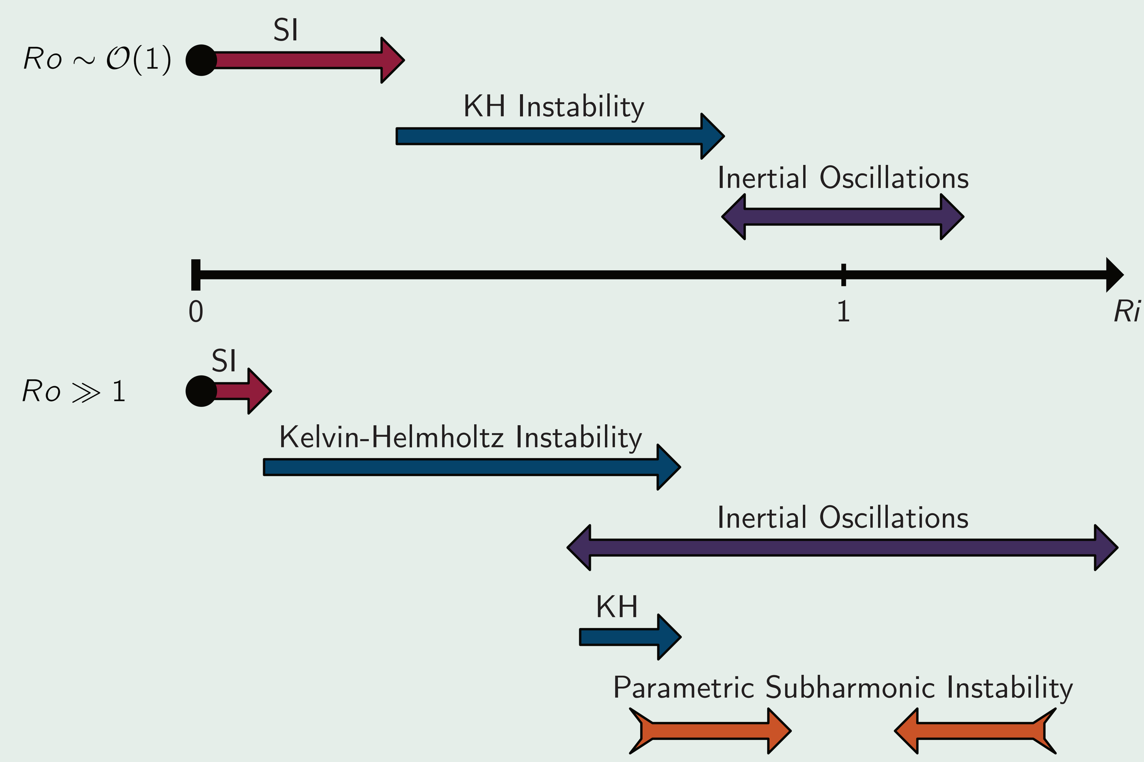
- The front loses geostrophic balance as the along-front shear is mixed down by SI and the resulting turbulent fluxes.
- The details of adjustment depend on the time-scales:
  - At **Small** Ro, inertial adjustments occur faster than SI growth and turbulent fluxes $\Longrightarrow$ Quasi-balanced adjustment
  - At **Large** Ro, SI & turbulence evolve rapidly before rotation effects influence the dynamics $\Longrightarrow$ Geostrophic adjustment



- The available PE required to be dissipated by turbulence to reach an SI-stable equilibration is $\frac{1}{3}Ro^2$.
  $\Longrightarrow$ More energetic turbulence at large Ro
- At the same time, the turbulence decay time, $\tau_d f \propto Ro^{-1}$, is shorter than the inertial period for large Ro.
  $\Longrightarrow$ Bursty behaviour during weakly stratified phases

6 Conclusions

- Momentum and buoyancy transport by the linear modes influence characteristics of the resulting adjustment.
- Lifetime and ultimate fate of submesoscale vertical fronts:



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